Chiral behavior of the $B_{d,s}^0 - \overline{B}_{d,s}^0$ mixing amplitude in the Standard Model and beyond

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ABSTRACT: We compute the chiral logarithmic corrections to the $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixing amplitudes in the Standard Model and beyond. We then investigate the impact of the inclusion of the lowest-lying scalar heavy-light states to the decay constants and bag-parameters and show that this **does not** modify the pion chiral logarithms, but it does produce corrections which are competitive in size with the K- and η-meson chiral logarithms. This conclusion is highly relevant to the lattice studies since the pion chiral logarithms represent the most important effect in guiding the chiral extrapolations of the lattice data for these quantities. It is also important to stress that the pion chiral logarithmic corrections are useful in guiding those chiral extrapolations as long as $m_\pi \ll \Delta_S$, where Δ_S stands for the mass difference between the heavy-light mesons belonging to $\frac{1}{2}^+$ and $\frac{1}{2}^-$ doublets.

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Contents

1. Introduction

The oscillations in the $B_{d,s}^0 - \overline{B}_{d,s}^0$ systems are mediated by the flavor changing neutral currents which are forbidden at tree level of the Standard Model (SM) and therefore their detection gives access to the particle content in the corresponding loop diagrams. First experimental measurement of a *large* value of Δm_{B_d} indicated that the top quark mass was very heavy [1], which was confirmed almost a decade later in the direct measurements, $m_t = 172.5 \pm 1.3 \pm 1.9$ GeV through the $p\bar{p}$-collisions [2]. Nowadays, the accurately measured $\Delta m_{B_d} = 0.509(5)(3)$ ps^{-1} [3], and $\Delta m_{B_s} = 17.31\binom{33}{17}(7)$ ps^{-1} [4], are used to constrain the shape of the Cabbibo-Kobayashi-Maskawa (CKM) unitarity triangle and thereby determine the amount of the CP-violation in the SM [5]. This goal is somewhat hampered by the theoretical uncertainties in computing the values for the two decay constants, $f_{B_{s,d}}$, and the corresponding "bag" parameters, $B_{B_{s,d}}$. These quantities can, in principle, be computed on the lattice.[1] However, a major obstacle in the current lattice studies is that the d-quark cannot be reached directly but through an extrapolation of the results obtained by working with larger light quark masses down to the physical d-quark mass. This extrapolation induces systematic uncertainties which are hard to control as the spontaneous chiral symmetry breaking effects are expected to become increasingly pronounced as one lowers the light quark mass [7]. Heavy meson chiral perturbation theory (HMChPT) allows us to gain some control over these uncertainties because it predicts the chiral behavior of the hadronic quantities relevant to the heavy-light quark phenomenology which then can

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[1]Recent reviews on the current status of the lattice QCD computations of $B_q^0 - \overline{B}_q^0$ mixing amplitudes can be found in ref. [6].

be implemented to guide the extrapolation of the lattice results. HMChPT combines the heavy quark effective theory (HQET) with the common pattern of spontaneous breaking of the chiral symmetry, $SU(3)_L \otimes SU(3)_R \to SU(3)_V$ [8].

Like in the standard ChPT, in HMChPT one computes the chiral logarithmic corrections (the so-called non-analytic terms) which are expected to be relevant to the very low energy region, i.e., $m_q \ll \Lambda_{QCD}$. While this condition is satisfied for u- and d-quarks, the situation with the s-quark is still unclear [9]. Also ambiguous is the size of the chiral symmetry breaking scale, Λ_χ. Some authors consider it to be around $4\pi f_\pi \simeq 1\,GeV$ [10], while others prefer identifying it with the mass of the first vector resonance, $m_\rho = 0.77\,GeV$ (see e.g. ref. [11]), and sometimes even lower [12]. In the heavy-light quark systems the situation becomes more complicated because the first orbital excitations ($j_\ell^P = 1/2^+$) are not far away from the lowest lying states ($j_\ell^P = 1/2^-$). The recent experimental evidence for the scalar D_{0s}^* and axial D_{1s} mesons indicate that this splitting is only $\Delta_{S_s} \equiv m_{D_{0s}^*} - m_{D_s} = m_{D_{1s}^*} - m_{D_s^*} = 350\,MeV$ [13], and somewhat larger for the non-strange states $\Delta_{S_{u,d}} = 430(30)\,MeV$ [14].[2] This and the result of the lattice QCD study in the static heavy quark limit [17] suggest that the size of this mass difference remains as such in the b-quark sector as well. One immediately observes that both Δ_{S_s} and $\Delta_{S_{u,d}}$ are smaller than Λ_χ, m_η, and even m_K, which requires revisiting the predictions based on HMChPT and reassessing their range of validity. In this paper we investigate this issue on the specific examples of the decay constants $f_{B_{d,s}}$ and the bag parameters which enter the investigation of the SM and supersymmetric (SUSY) effects in the $B_d^0 - \overline{B}_d^0$ and $B_s^0 - \overline{B}_s^0$ mixing amplitudes [18].

2. Bases of $\Delta B = 2$ operators and B-parameters

The SUSY contributions to the $B_q^0 - \overline{B}_q^0$ mixing amplitude, where q stands for either d- or s-quark, are usually discussed in the so called SUSY basis of $\Delta B = 2$ operators [19]:

$$
\begin{aligned}
O_1 &= \bar{b}^i \gamma_\mu (1 - \gamma_5) q^i\, \bar{b}^j \gamma^\mu (1 - \gamma_5) q^j\,, \\
O_2 &= \bar{b}^i (1 - \gamma_5) q^i\, \bar{b}^j (1 - \gamma_5) q^j\,, \\
O_3 &= \bar{b}^i (1 - \gamma_5) q^j\, \bar{b}^j (1 - \gamma_5) q^i\,, \\
O_4 &= \bar{b}^i (1 - \gamma_5) q^i\, \bar{b}^j (1 + \gamma_5) q^j\,, \\
O_5 &= \bar{b}^i (1 - \gamma_5) q^j\, \bar{b}^j (1 + \gamma_5) q^i\,,
\end{aligned}
\tag{2.1}
$$

where i and j are the color indices. Although the operators in the above bases are written with both parity even and parity odd parts, only the parity even ones survive in the matrix elements. In SM, only O_1 (left-left) operator is relevant in describing the $B_q^0 - \overline{B}_q^0$ mixing amplitude. The matrix elements of the above operators are conventionally parameterized

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[2]We note, in passing, that the experimentally established fact, $\Delta_{S_s} < \Delta_{S_{u,d}}$, is not yet understood [15] although a recent lattice study with the domain wall quarks indicates a qualitative agreement with experiment [16].

in terms of bag-parameters, B_{1-5}, as a measure of the discrepancy with respect to the estimate obtained by using the vacuum saturation approximation (VSA),

$$\frac{\langle \bar{B}_q^0|O_{1-5}(\nu)|B_q^0\rangle}{\langle \bar{B}_q^0|O_{1-5}(\nu)|B_q^0\rangle_{\text{VSA}}} = B_{1-5}(\nu), \quad (2.2)$$

where ν is the renormalisation scale of the logarithmically divergent operators, O_i, at which the separation between the long-distance (matrix elements) and short-distance (Wilson coefficients) physics is made. We remind the reader that

$$\langle \bar{B}_q^0|O_1|B_q^0\rangle_{\text{VSA}} = 2\left(1 + \frac{1}{3}\right)\langle \bar{B}_q^0|A_\mu|0\rangle\,\langle 0|A^\mu|B_q^0\rangle,$$

$$\langle \bar{B}_q^0|O_2|B_q^0\rangle_{\text{VSA}} = -2\left(1 - \frac{1}{6}\right)\left|\langle 0|P|B_q^0\rangle\right|^2,$$

$$\langle \bar{B}_q^0|O_3|B_q^0\rangle_{\text{VSA}} = \left(1 - \frac{2}{3}\right)\left|\langle 0|P|B_q^0\rangle\right|^2, \quad (2.3)$$

$$\langle \bar{B}_q^0|O_4|B_q^0\rangle_{\text{VSA}} = \frac{1}{3}\langle \bar{B}_q^0|A_\mu|0\rangle\,\langle 0|A^\mu|B_q^0\rangle + 2\left|\langle 0|P|B_q^0\rangle\right|^2,$$

$$\langle \bar{B}_q^0|O_5|B_q^0\rangle_{\text{VSA}} = \langle \bar{B}_q^0|A_\mu|0\rangle\,\langle 0|A^\mu|B_q^0\rangle + \frac{2}{3}\left|\langle 0|P|B_q^0\rangle\right|^2,$$

with $A_\mu = \bar{b}\gamma_\mu\gamma_5 q$ and $P = \bar{b}\gamma_5 q$ being the axial current and the pseudoscalar density, respectively. In HQET, in which we will be working from now on, the field $\bar{b}$ is replaced by the static one, $h^\dagger$, which satisfies $h^\dagger\gamma_0 = h^\dagger$. This equation and the fact that the amplitude is invariant under the Fierz transformation in Dirac indices, eliminate the operator O_3 from further discussion, i.e., $\langle \bar{B}_q^0|\widetilde{O}_3 + \widetilde{O}_2 + \frac{1}{2}\widetilde{O}_1|B_q^0\rangle = 0$, where the tilde is used to stress that the operators are now being considered in the static limit of HQET ($|\vec{v}| = 0$). Furthermore, in the same limit

$$\lim_{m_b\to\infty}\frac{\langle 0|A_\mu|B_q^0(p)\rangle_{\text{QCD}}}{\sqrt{2m_B}} = \lim_{m_b\to\infty}\frac{\langle 0|P|B_q^0(p)\rangle_{\text{QCD}}}{\sqrt{2m_B}} = \langle 0|\widetilde{A}_0|B_q^0(v)\rangle_{\text{HQET}} = i\hat{f}_q, \quad (2.4)$$

where $\hat{f}_q$ is the decay constant of the static $1/2^-$ heavy-light meson, and the HQET states are normalized as $\langle B_q^0(v)|B_q^0(v')\rangle = \delta(v - v')$, so that we finally have

$$\langle \bar{B}_q^0|\widetilde{O}_1(\nu)|B_q^0\rangle = \frac{8}{3}\hat{f}_q(\nu)^2\widetilde{B}_{1q}(\nu),$$

$$\langle \bar{B}_q^0|\widetilde{O}_2(\nu)|B_q^0\rangle = -\frac{5}{3}\hat{f}_q(\nu)^2\widetilde{B}_{2q}(\nu), \quad (2.5)$$

$$\langle \bar{B}_q^0|\widetilde{O}_4(\nu)|B_q^0\rangle = \frac{7}{3}\hat{f}_q(\nu)^2\widetilde{B}_{4q}(\nu),$$

$$\langle \bar{B}_q^0|\widetilde{O}_5(\nu)|B_q^0\rangle = \frac{5}{3}\hat{f}_q(\nu)^2\widetilde{B}_{5q}(\nu).$$

One of the reasons why lattice QCD is the best currently available method for computing these matrix elements is the fact that it enables a control over the ν-dependence by verifying the corresponding renormalisation group equations, which is essential for the cancellation against the ν-dependence in the corresponding perturbatively computed Wilson coefficients [20]. From now on we will assume that the UV divergences are being taken care of and the scale ν will be implicit.

3. Chiral logarithmic corrections

In this section we use HMChPT to describe the low energy behavior of the matrix elements (2.5). Before entering the details, we notice that the operators $\widetilde{O}_4$ and $\widetilde{O}_5$ differ only in the color indices, i.e., by a gluon exchange, which is a local effect that cannot influence the long distance behavior described by ChPT. In other words, from the point of view of ChPT, the entire difference of the chiral behavior of the bag parameters $\widetilde{B}_{4q}$ and $\widetilde{B}_{5q}$ is encoded in the local counter-terms, whereas their chiral logarithmic behavior is the same. Similar observation has been made for the operators entering the SUSY analysis of the $\bar{K}^0$-K^0 mixing amplitude, as well as for the electromagnetic penguin operators in $K \to \pi\pi$ decay [21]. Thus, in the static heavy quark limit ($m_Q \to \infty$), we are left with the first three operators in eq. (2.5) which, in their bosonised version, can be written as [22][3]

$$\widetilde{O}_1 = \sum_X \beta_{1X}\mathrm{Tr}\left[(\xi\overline{H}^Q)_q\gamma_\mu(1-\gamma_5)X\right]\mathrm{Tr}\left[(\xi H^{\bar{Q}})_q\gamma^\mu(1-\gamma_5)X\right] + \text{c.t.},$$

$$\widetilde{O}_2 = \sum_X \beta_{2X}\mathrm{Tr}\left[(\xi\overline{H}^Q)_q(1-\gamma_5)X\right]\mathrm{Tr}\left[(\xi H^{\bar{Q}})_q(1-\gamma_5)X\right] + \text{c.t.},$$

$$\widetilde{O}_4 = \sum_X \beta_{4X}\mathrm{Tr}\left[(\xi\overline{H}^Q)_q(1-\gamma_5)X\right]\mathrm{Tr}\left[(\xi^\dagger H^{\bar{Q}})_q(1+\gamma_5)X\right]$$
$$+ \bar{\beta}_{4X}\mathrm{Tr}\left[(\xi H^{\bar{Q}})_q(1-\gamma_5)X\right]\mathrm{Tr}\left[(\xi^\dagger\overline{H}^Q)_q(1+\gamma_5)X\right] + \text{c.t.}, \qquad (3.1)$$

where $X \in \{1, \gamma_5, \gamma_\nu, \gamma_\nu\gamma_5, \sigma_{\nu\rho}\}$.[4] As before the index "q" denotes the light quark flavor, and "c.t." stands for the local counter-terms. To relate β_i's to the bag parameters in eq. (2.5) we should recall that the field H_q is built up from the pseudoscalar (P) and the vector (P^*) meson fields as

$$H_q^Q(v) = \frac{1+\not{v}}{2}\left[P_\mu^{Q*}(v)\gamma^\mu - P^Q(v)\gamma_5\right]_q,$$
$$H_q^{\bar{Q}}(v) = \left[P_\mu^{\bar{Q}*}(v)\gamma^\mu - P^{\bar{Q}}(v)\gamma_5\right]_q\frac{1-\not{v}}{2}, \qquad (3.2)$$

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[3]In the first version of the present paper only the factorisable pieces in the bosonised forms of the operators $\widetilde{O}_{2,4}$ were considered. We thank David Lin for pointing out to us the presence of the corresponding nonfactorisable pieces, recently considered in ref. [22], which are being properly accounted for in this version of our paper.

[4]Contraction of Lorentz indices and HQET parity conservation requires the same X to appear in both traces of a summation term. Any insertions of $\not{v}$ can be absorbed via $\not{v}H = H$, while any non-factorizable contribution with a single trace over Dirac matrices can be reduced to this form by using the 4×4 matrix identity

$$4\mathrm{Tr}(AB) = \mathrm{Tr}(A)\mathrm{Tr}(B) + \mathrm{Tr}(\gamma_5 A)\mathrm{Tr}(\gamma_5 B) + \mathrm{Tr}(A\gamma_\mu)\mathrm{Tr}(\gamma^\mu B)$$

$$+ \mathrm{Tr}(A\gamma_\mu\gamma_5)\mathrm{Tr}(\gamma_5\gamma^\mu B) + 1/2\mathrm{Tr}(A\sigma_{\mu\nu})\mathrm{Tr}(\sigma^{\mu\nu}B).$$

thus exhibiting the heavy quark spin symmetry for the lowest lying $j_\ell^P = 1/2^-$ states. After evaluating the traces in eq. (3.1) we obtain

$$\widetilde{B}_1 = \frac{3}{2\hat{f}^2}\widehat{\beta}_1\,, \quad \widetilde{B}_2 = \frac{12}{5\hat{f}^2}\widehat{\beta}_2\,, \quad \widetilde{B}_4 = \frac{12}{7\hat{f}^2}\widehat{\beta}_4\,, \quad \widetilde{B}_5 = \frac{12}{5\hat{f}^2}\widehat{\beta}_4\,, \tag{3.3}$$

where

$$\begin{aligned}
\widehat{\beta}_1 &= \beta_1 + \beta_{1\gamma_5} - 4(\beta_{1\gamma_\nu} + \beta_{1\gamma_\nu\gamma_5}) - 12\beta_{1\sigma_{\nu\rho}}, \\
\widehat{\beta}_2 &= -\beta_2 - \beta_{2\gamma_5} + \beta_{2\gamma_\nu} + \beta_{2\gamma_\nu\gamma_5}, \\
\widehat{\beta}_4 &= \beta_4 - \beta_{4\gamma_5} - \beta_{4\gamma_\nu} + \beta_{4\gamma_\nu\gamma_5} + \bar{\beta}_4 - \bar{\beta}_{4\gamma_5} - \bar{\beta}_{4\gamma_\nu} + \bar{\beta}_{4\gamma_\nu\gamma_5}.
\end{aligned} \tag{3.4}$$

We will use the well known form of the HMChPT lagrangian [8]

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{\text{light}} + \mathcal{L}_{\frac{1}{2}^-} + \mathcal{L}_{\text{ct.}}\,, \\
\mathcal{L}_{\text{light}} &= \frac{f^2}{8}\text{tr}\left[(\partial_\mu \Sigma^\dagger)(\partial^\mu \Sigma) + \Sigma^\dagger \chi + \chi^\dagger \Sigma\right]\,, \\
\mathcal{L}_{\frac{1}{2}^-} &= i\text{Tr}\left[H_b v\cdot D_{ba}\overline{H}_a\right] + g\text{Tr}\left[H_b \gamma_\mu \gamma_5 \mathbf{A}_{ba}^\mu \overline{H}_a\right]\,, \\
\mathcal{L}_{\text{ct}} &= k_1 \text{Tr}\left[\overline{H}_a H_b \left(\xi \mathcal{M}\xi + \xi^\dagger \mathcal{M}\xi^\dagger\right)_{ba}\right] + k_2 \text{Tr}\left[\overline{H}_a H_a \left(\xi \mathcal{M}\xi + \xi^\dagger \mathcal{M}\xi^\dagger\right)_{bb}\right]\,, \quad (3.5)
\end{aligned}$$

where $\overline{H}_a(v) = \gamma_0 H_a^\dagger(v)\gamma_0$, g is the coupling of the pseudo-Goldstone boson to the pair of heavy-light mesons, and

$$D_{ba}^\mu H_b = \partial^\mu H_a - H_b \frac{1}{2}[\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger]_{ba}\,, \quad \mathbf{A}_\mu^{ab} = \frac{i}{2}[\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger]_{ab}\,,$$

$$\xi = \sqrt{\Sigma}\,, \qquad \Sigma = \exp\left(2i\frac{\phi}{f}\right)\,, \qquad \mathcal{M} = \text{diag}(m_u, m_d, m_s)\,,$$

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}\,, \tag{3.6}$$

with $f \approx 130\,\text{MeV}$, and $\chi = 2B_0\mathcal{M}$. In the above formulae H_a refers to either H_a^Q or $H_a^{\bar{Q}}$, defined in eq. (3.2). Note also that we distinguish between the trace over Dirac ("Tr") and flavor ("tr") indices. In the chiral power counting the lagrangian $\mathcal{L}_{\text{light}}$ in eq. (3.5) is of $\mathcal{O}(p^2)$ while the rest of $\mathcal{L}$ is of $\mathcal{O}(p^1)$. To get the chiral logarithmic corrections to $\widetilde{B}_{iq}$-parameters, we should subtract twice the chiral corrections to the decay constant $\hat{f}_q$ from the chiral corrections to the four-quark operators (3.1). The former is obtained from the study of the bosonised left-handed weak current

$$(V - A)_q^\mu = \frac{i\alpha}{2}\left\{\text{Tr}\left[(\xi\overline{H})_q \gamma^\mu (1 - \gamma_5)\right](1 + \varkappa_2 \text{tr}\mathcal{M}) + \varkappa_1 \text{Tr}\left[(\xi\mathcal{M}\overline{H})_q \gamma^\mu (1 - \gamma_5)\right]\right\}\,, \tag{3.7}$$

where α is the tree level decay constant in the chiral expansion, and $\varkappa_{1,2}$ are the counter-term coefficients. Together with the strong coupling g, these parameters are not predicted
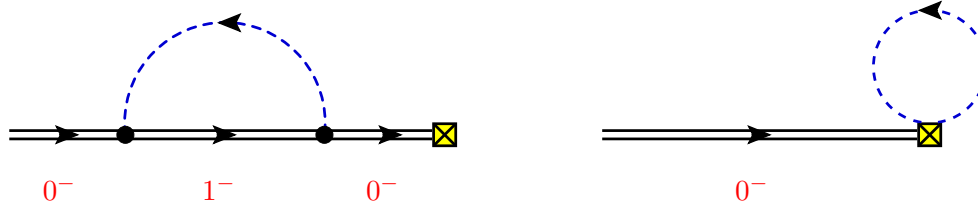
Figure 1: The diagrams which give non-vanishing chiral logarithmic corrections to the pseudoscalar heavy-light meson decay constant. The double line indicates the heavy-light meson and the dashed one the pseudo-Goldstone boson propagator. The square stands for the weak current vertex. The full dot is proportional to the coupling g.

within HMChPT. Instead, they are expected to be fixed by matching the HMChPT expressions with the results of lattice QCD for a given quantity (see reviews in ref. [23]). The notation used above is the same as in ref. [24].

The chiral logarithmic corrections to the decay constant come from the diagrams shown in figure 1

$$\hat{f}_d = \alpha \left[1 - \frac{1}{(4\pi f)^2} \left(\frac{3}{4} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + \frac{1}{2} m_K^2 \log \frac{m_K^2}{\mu^2} + \frac{1}{12} m_\eta^2 \log \frac{m_\eta^2}{\mu^2} \right) \right.$$

$$\left. + \varkappa_1(\mu) m_d + \varkappa_2(\mu)(m_u + m_d + m_s) + \frac{1}{2} \delta Z_d \right],$$

$$\hat{f}_s = \alpha \left[1 - \frac{1}{(4\pi f)^2} \left(m_K^2 \log \frac{m_K^2}{\mu^2} + \frac{1}{3} m_\eta^2 \log \frac{m_\eta^2}{\mu^2} \right) \right. \tag{3.8}$$

$$\left. + \varkappa_1(\mu) m_s + \varkappa_2(\mu)(m_u + m_d + m_s) + \frac{1}{2} \delta Z_s \right],$$

where it should be stressed that we work in the exact isospin limit ($m_u = m_d$) so that the index d means either u- or d-quark. Only explicit in the above expressions is the term arising from the tadpole diagram (right in figure 1), whereas $Z_{d,s}$, the heavy meson field renormalization factors, come from the self energy diagram (left in figure 1) and they read

$$Z_d = 1 - \frac{3g^2}{(4\pi f)^2} \left(\frac{3}{2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + m_K^2 \log \frac{m_K^2}{\mu^2} + \frac{1}{6} m_\eta^2 \log \frac{m_\eta^2}{\mu^2} \right)$$

$$+ k_1(\mu) m_d + k_2(\mu)(m_u + m_d + m_s), \tag{3.9}$$

$$Z_s = 1 - \frac{3g^2}{(4\pi f)^2} \left(2m_K^2 \log \frac{m_K^2}{\mu^2} + \frac{2}{3} m_\eta^2 \log \frac{m_\eta^2}{\mu^2} \right) + k_1(\mu) m_s + k_2(\mu)(m_u + m_d + m_s) .$$

In both eqs. (3.8) and (3.9) the μ dependence in the logarithm cancels against the one in the local counter-terms.

With these ingredients in hands it is now easy to deduce that the only diagrams which contribute to the SM bag parameter, $\widetilde{B}_{1q}$, are the two shown in figure 2. They arise from
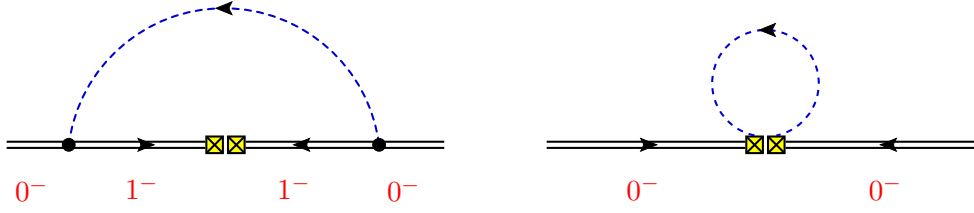
Figure 2: The diagrams relevant to the chiral corrections to the SM bag parameter $\widetilde{B}_{1q}$. In the text we refer to the left one as "sunset", and to the right one as "tadpole". Only the tadpole diagram gives a non-vanishing contribution to the bag parameters $\widetilde{B}_{2,4q}$.

the two terms in $\widetilde{O}_1 = 4\widehat{\beta}_1[(\xi\bar{P}_\mu^*)_q(\xi\bar{P}^{*\mu})_q + (\xi\bar{P})_q(\xi\bar{P})_q]$ and yield

$$\text{"sunset"}: \quad 4\widehat{\beta}_1 \frac{3g^2}{(4\pi f)^2} \sum_i (t_{qq}^i)^2 m_i^2 \log\frac{m_i^2}{\mu^2},$$

$$\text{"tadpole"}: -4\widehat{\beta}_1 \frac{1}{(4\pi f)^2} \sum_i (t_{qq}^i)^2 m_i^2 \log\frac{m_i^2}{\mu^2}, \tag{3.10}$$

respectively, where t^i are the SU(3) generators and m_i masses of the pseudo-Goldstone bosons. The SM bag parameters now read

$$\widetilde{B}_{1d} = \widetilde{B}_1^{\text{Tree}}\left[1 - \frac{1-3g^2}{(4\pi f)^2}\left(\frac{1}{2}m_\pi^2\log\frac{m_\pi^2}{\mu^2} + \frac{1}{6}m_\eta^2\log\frac{m_\eta^2}{\mu^2}\right)\right.$$
$$\left. + b_1(\mu)m_d + b_1'(\mu)(m_u + m_d + m_s)\right], \tag{3.11}$$

$$\widetilde{B}_{1s} = \widetilde{B}_1^{\text{Tree}}\left[1 - \frac{1-3g^2}{(4\pi f)^2}\frac{2}{3}m_\eta^2\log\frac{m_\eta^2}{\mu^2} + b_1(\mu)m_s + b_1'(\mu)(m_u + m_d + m_s)\right],$$

where we also wrote the counter-term contributions and, for short, we wrote $\widetilde{B}_1^{\text{Tree}} = 3\widehat{\beta}_1/2\alpha^2$. The above results agree with the ones presented in refs. [25, 26], in which the pion loop contribution was left out, and with the ones recently presented in ref. [27].

As for the bag parameters $\widetilde{B}_{2q}$ and $\widetilde{B}_{4q}$ we obtain

$$\widetilde{B}_{2,4d} = \widetilde{B}_{2,4}^{\text{Tree}}\left[1 + \frac{3g^2Y \mp 1}{(4\pi f)^2}\left(\frac{1}{2}m_\pi^2\log\frac{m_\pi^2}{\mu^2} + \frac{1}{6}m_\eta^2\log\frac{m_\eta^2}{\mu^2}\right)\right.$$
$$\left. + b_{2,4}(\mu)m_d + b_{2,4}'(\mu)(m_u + m_d + m_s)\right], \tag{3.12}$$

$$\widetilde{B}_{2,4s} = \widetilde{B}_{2,4}^{\text{Tree}}\left[1 + \frac{2}{3}\frac{3g^2Y \mp 1}{(4\pi f)^2}m_\eta^2\log\frac{m_\eta^2}{\mu^2} + b_{2,4}(\mu)m_s + b_{2,4}'(\mu)(m_u + m_d + m_s)\right],$$

where $\widetilde{B}_2^{\text{Tree}} = 12\widehat{\beta}_2/4\alpha^2$, $\widetilde{B}_4^{\text{Tree}} = 12\widehat{\beta}_2/7\alpha^2$ $Y = (\widehat{\beta}_{2,4}^*/\widehat{\beta}_{2,4})$, with $\widehat{\beta}_2^* = \beta_{2\gamma_\nu} + \beta_{2\gamma_\nu\gamma_5} + 4\beta_{2\sigma_{\nu\rho}}$, and $\widehat{\beta}_4^* = \beta_{4\gamma_\nu} - \beta_{4\gamma_\nu\gamma_5} + \bar{\beta}_{4\gamma_\nu} - \bar{\beta}_{4\gamma_\nu\gamma_5}$. We checked that our results agree with those presented in ref. [27] where also the partially quenched theory has been considered. In our paper we refer only to the full (unquenched) theory.

4. Impact of the $1/2^+$-mesons

In this section we examine the impact of the heavy-light mesons belonging to the $1/2^+$ doublet when propagating in the loops onto the chiral logarithmic corrections derived in the previous section. We first extend the lagrangian by adding to eq. (3.5) the following terms [8]:

$$
\begin{aligned}
\mathcal{L}_{\frac{1}{2}^+} &= -\mathrm{Tr}\left[S_b(iv\cdot D_{ba} + \Delta_S)\overline{S}_a\right] + \tilde{g}\,\mathrm{Tr}\left[S_b\gamma_\mu\gamma_5 \mathbf{A}_{ba}^\mu \overline{S}_a\right], \\
\mathcal{L}_{\mathrm{mix}} &= h\,\mathrm{Tr}\left[S_b\gamma_\mu\gamma_5 \mathbf{A}_{ba}^\mu \overline{H}_a\right] + \mathrm{h.c.}, \\
\mathcal{L}_{\mathrm{ct}} &= \widetilde{k}_1\,\mathrm{Tr}\left[\overline{S}_a S_b\left(\xi\mathcal{M}\xi + \xi^\dagger\mathcal{M}\xi^\dagger\right)_{ba}\right] + \widetilde{k}_2\,\mathrm{Tr}\left[\overline{S}_a S_a\left(\xi\mathcal{M}\xi + \xi^\dagger\mathcal{M}\xi^\dagger\right)_{bb}\right] \\
&\quad + k_1'\,\mathrm{Tr}\left[\overline{H}_a S_b\left(\xi\mathcal{M}\xi + \xi^\dagger\mathcal{M}\xi^\dagger\right)_{ba}\right] + k_2'\,\mathrm{Tr}\left[\overline{H}_a S_a\left(\xi\mathcal{M}\xi + \xi^\dagger\mathcal{M}\xi^\dagger\right)_{bb}\right] + \mathrm{h.c.},
\end{aligned} \tag{4.1}
$$

where the fields of the scalar (P_0) and the axial ($P_{1\ \mu}^*$) mesons are organised in a superfield

$$
S_q(v) = \frac{1+\slashed{v}}{2}\left[P_{1\ \mu}^*(v)\gamma_\mu\gamma_5 - P_0(v)\right]_q, \qquad \overline{S}_q(v) = \gamma_0 S_q^\dagger(v)\gamma_0. \tag{4.2}
$$

$\tilde{g}$ is the coupling of the P-wave Goldstone boson to the pair of $1/2^+$ heavy-light mesons, and h is the coupling of the S-wave Goldstone boson to the heavy-light mesons, one of which belongs to $1/2^-$ and the other to $1/2^+$ doublet. Before including the $1/2^+$ doublet we were free to set $\Delta = 0$ because all the chiral loop divergences are cancelled by $\mathcal{O}(m_q)$ counterterms in the static heavy quark limit. Once the $1/2^+$ doublet is included, the mass difference between the $1/2^+$ and $1/2^-$ states ($\Delta_S \approx 400\,\mathrm{MeV}$) must be included in the lagrangian, but since it does not vanish in the chiral nor in the heavy quark limit it is of $\mathcal{O}(p^0)$ in the chiral power counting (see also ref. [28] where, in addition to the static heavy quark limit, the chiral power counting is discussed also when the $1/m_Q$-corrections are included).

4.1 Decay constants

Beside the lagrangian, the $1/2^+$ mesons contribution should also be added to the left vector current (3.7), which now reads

$$
\begin{aligned}
(V-A)_q^\mu &= \frac{i\alpha}{2}\mathrm{Tr}\left[(\xi\overline{H})_q\gamma^\mu(1-\gamma_5)\right] + \frac{i\alpha^+}{2}\mathrm{Tr}\left[(\xi\overline{S})_q\gamma^\mu(1-\gamma_5)\right] \\
&\quad + \frac{i\alpha}{2}\varkappa_1\mathrm{Tr}\left[(\xi\mathcal{M}\overline{H})_q\gamma^\mu(1-\gamma_5)\right] + \frac{i\alpha^+}{2}\widetilde{\varkappa}_1\mathrm{Tr}\left[(\xi\mathcal{M}\overline{S})_q\gamma^\mu(1-\gamma_5)\right] \\
&\quad + \frac{i\alpha}{2}\varkappa_2\mathrm{Tr}\left[(\xi\overline{H})_q\gamma^\mu(1-\gamma_5)\right]\mathrm{tr}\mathcal{M} + \frac{i\alpha^+}{2}\widetilde{\varkappa}_2\mathrm{Tr}\left[(\xi\overline{S})_q\gamma^\mu(1-\gamma_5)\right]\mathrm{tr}\mathcal{M},
\end{aligned} \tag{4.3}
$$

where α^+ is the coupling of one of the $1/2^+$ mesons to the weak left current, and $\widetilde{\varkappa}_{1,2}$ are the coefficients of two new counter-terms. From the recent lattice results reported in ref. [29] we extract, $\alpha^+/\alpha = 1.1(2)$. In other words, at least in the static heavy quark mass limit, the weak current coupling of $1/2^+$ mesons is not suppressed with respect to the $1/2^-$ ones. Since we focus on the pseudoscalar meson decay constant, it should be clear that only the scalar meson from the $1/2^+$ doublet can propagate in the loop. The diagrams that
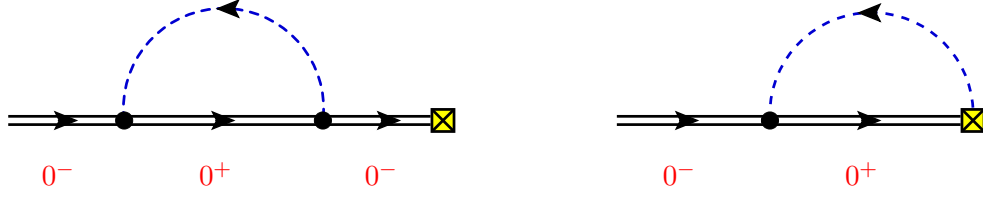
Figure 3: In addition to the diagrams shown in figure 1, these two diagrams contribute the loop corrections to the pseudoscalar meson decay constant after the $1/2^+$ mesons are included in HMChPT. The full dot in these graphs is proportional to the coupling h.

give non-vanishing contributions are shown in figure 3 and the corresponding expressions now read

$$Z_q = 1 + \frac{t^i_{qa} t^{i\dagger}_{aq}}{(4\pi f)^2} \left\{ 3g^2 \lim_{x\to 0} \frac{d}{dx}[x J_1(m_i^2, x)] - h^2 \Big[J_1(m_i^2, \Delta_S) + J_2(m_i^2, \Delta_S) \right. \tag{4.4}$$
$$\left. + \Delta_S \frac{d}{d\Delta_S} \big(J_1(m_i^2, \Delta_S) + J_2(m_i^2, \Delta_S) \big) \Big] \right\},$$

$$\hat{f}_q = \alpha \left\{ 1 + \frac{t^i_{qa} t^{i\dagger}_{aq}}{2(4\pi f)^2} \left[3g^2 \lim_{x\to 0} \frac{d}{dx}[x J_1(m_i^2, x)] - I_1(m_i^2) - h^2 \Big(J_1(m_i^2, \Delta_S) + J_2(m_i^2, \Delta_S) \right.\right.$$
$$\left.\left. + \Delta_S \frac{d}{d\Delta_S} \big(J_1(m_i^2, \Delta_S) + J_2(m_i^2, \Delta_S) \big) \Big) - 2h \frac{\alpha^+}{\alpha} \big(I_1(m_i^2) + I_2(m_i^2, \Delta_S) \big) \right] \right\},$$

where the summation over "i" is implicit, and we omit the counter-term contributions to make the expressions simpler. We stress that in all our formulae, the terms of $\mathcal{O}(p^3)$ and higher in the chiral power counting are neglected. The integrals $I_{1,2}$ and $J_{1,2}$ are the same as the ones used in ref. [24] and can be found in the appendix of that paper. The terms proportional to h^2 come from the inclusion of the left diagram shown in figure 3, while the last term in the decay constant (the one proportional to h) comes from the right graph in figure 3. Obviously such terms were absent before including the scalar meson. Notice also that

$$\lim_{x\to 0} \frac{d}{dx}[x J_1(m_\pi^2, x)] = -m_\pi^2 \log \frac{m_\pi^2}{\mu^2}. \tag{4.5}$$

When $\Delta_S > m_i$, which in our case is true for the pion mass, we can expand around $m_\pi^2 = 0$ and obtain

$$J_1(m_\pi^2, \Delta_S) + J_2(m_\pi^2, \Delta_S) =$$
$$I_1(m_\pi^2) + I_2(m_\pi^2, \Delta_S) \to 2\Delta_S^2 \left(1 - \log \frac{4\Delta_S^2}{\mu^2} \right) + m_\pi^2 \left(1 + \log \frac{4\Delta_S^2}{\mu^2} \right) + \dots,$$
$$-\Delta_S \frac{d}{d\Delta_S} [J_1(m_\pi^2, \Delta_S) + J_2(m_\pi^2, \Delta_S)] \to 4\Delta_S^2 \log \frac{4\Delta_S^2}{\mu^2} - 2m_\pi^2 + \dots, \tag{4.6}$$

where the dots stand for higher powers in m_π^2. In other words, the presence of the nearby $1/2^+$ state does not affect the pionic logarithmic behavior of the decay constant. It does,

however, affect the kaon and η-meson loops because those states are heavier than Δ_S ($m_\pi < \Delta_S \lesssim m_K < m_\eta$) and the coefficients of their logarithms, although still predictions of this approach, cease to be numerically relevant because those logarithms are competitive in size with the terms proportional to $\Delta_S^2 \log(4\Delta_S^2/\mu^2)$, as indicated in eq. (4.6). Stated equivalently, the relevant chiral logarithmic corrections are those coming from the $SU(2)_L \otimes SU(2)_R \to SU(2)_V$ theory, and the pseudoscalar decay constant reads

$$\hat{f}_q = \alpha \left[1 - \frac{1 + 3g^2}{2(4\pi f)^2} \frac{3}{2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_f(\mu) m_\pi^2 \right], \tag{4.7}$$

where $c_f(\mu)$ stands for the combination of the counter-term coefficients considered in the previous section.[5] At this point we also note that we checked that the chiral logarithms in the scalar heavy-light meson decay constant, which has recently been computed on the lattice in ref. [29], are the same as for the pseudoscalar meson, with the coupling g being replaced by $\widetilde{g}$, i.e.,

$$\hat{f}_q^+ = \alpha^+ \left[1 - \frac{1 + 3\widetilde{g}^2}{2(4\pi f)^2} \frac{3}{2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_f^+(\mu) m_\pi^2 \right]. \tag{4.8}$$

Since $\widetilde{g}^2/g^2 \approx 1/9$ [30], the deviation from the linear behavior in m_π^2 is less pronounced for $\hat{f}_q^+$ than it is for $\hat{f}_q$. Finally, it should be emphasized that the counterterm coefficients relevant to the $SU(2)_V$-theory, namely $c_f^{(+)}(\mu)$ in eqs. (4.7), (4.8), are not the same as those in $SU(3)_V$.

4.2 Bag parameters

In this subsection we show that the situation with the bag parameters is similar to the one with decay constant, namely the pion loop chiral logarithmis remain unchanged when the nearby scalar meson is included in HMChPT. To that end, besides eq. (4.1), we should include the contributions of $1/2^+$-mesons to the operators (3.1). Generically the operators $\widetilde{\mathcal{O}}_{1,2,4}$ now become

$$\widetilde{O}_1 = \sum_X \beta_{1X} \mathrm{Tr} \left[\left(\xi \overline{H}^Q \right)_q \gamma_\mu (1 - \gamma_5) X \right] \mathrm{Tr} \left[\left(\xi H^{\bar{Q}} \right)_q \gamma^\mu (1 - \gamma_5) X \right]$$

$$+ \beta'_{1X} \left\{ \mathrm{Tr} \left[\left(\xi \overline{H}^Q \right)_q \gamma_\mu (1 - \gamma_5) X \right] \mathrm{Tr} \left[\left(\xi S^{\bar{Q}} \right)_q \gamma^\mu (1 - \gamma_5) X \right] + \mathrm{h.c.} \right\}$$

$$+ \beta''_{1X} \mathrm{Tr} \left[\left(\xi \overline{S}^Q \right)_q \gamma_\mu (1 - \gamma_5) X \right] \mathrm{Tr} \left[\left(\xi S^{\bar{Q}} \right)_q \gamma^\mu (1 - \gamma_5) X \right], \tag{4.9}$$

---

[5]More specifically, $2B_0 c_f(\mu) + \frac{3h^2}{4(4\pi f)^2} \left[3 + \log(4\Delta_S^2/\mu^2) \right] + \frac{3h\alpha^+}{2\alpha(4\pi f)^2} [1 + \log(4\Delta_S^2/\mu^2)] = \frac{1}{2} k_1(\mu) + \frac{1}{2} k'_1(\mu) + k_2(\mu) + k'_2(\mu) + \varkappa_1(\mu) + 2\varkappa_2(\mu)$, where we use the Gell-Mann-Oakes-Renner formula, $m_\pi^2 = 2B_0 m_d$. The exact isospin symmetry ($m_u = m_d$) is assumed throughout this work.

where β'_{1X} are the couplings of the operator $\widetilde{O}_1$ to both $1/2^-$ and $1/2^+$ mesons, while β''_{1X} come from the coupling to the $1/2^+$ mesons only. Similarly, the operators $\widetilde{O}_{2,4}$ now read:

$$
\begin{aligned}
\widetilde{O}_2 = \sum_X \beta_{2X} &\mathrm{Tr}\left[\left(\xi\overline{H}^Q\right)_q (1-\gamma_5)X\right] \mathrm{Tr}\left[\left(\xi H^{\bar{Q}}\right)_q (1-\gamma_5)X\right] \\
&+\beta'_{2X}\left\{\mathrm{Tr}\left[\left(\xi\overline{H}^Q\right)_q (1-\gamma_5)X\right] \mathrm{Tr}\left[\left(\xi S^{\bar{Q}}\right)_q (1-\gamma_5)X\right] + \mathrm{h.c.}\right\} \\
&+\beta''_{2X}\mathrm{Tr}\left[\left(\xi\overline{S}^Q\right)_q (1-\gamma_5)X\right] \mathrm{Tr}\left[\left(\xi S^{\bar{Q}}\right)_q (1-\gamma_5)X\right] , \quad (4.10)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{O}_4 = \sum_X \beta_{4X} &\mathrm{Tr}\left[\left(\xi\overline{H}^Q\right)_q (1-\gamma_5)X\right] \mathrm{Tr}\left[\left(\xi^\dagger H^{\bar{Q}}\right)_q (1+\gamma_5)X\right] \\
&+\bar{\beta}_{4X}\mathrm{Tr}\left[\left(\xi H^{\bar{Q}}\right)_q (1-\gamma_5)X\right] \mathrm{Tr}\left[\left(\xi^\dagger \overline{H}^Q\right)_q (1+\gamma_5)X\right] \\
&+\beta'_{4X}\left\{\mathrm{Tr}\left[\left(\xi\overline{H}^Q\right)_q (1-\gamma_5)X\right] \mathrm{Tr}\left[\left(\xi^\dagger S^{\bar{Q}}\right)_q (1+\gamma_5)X\right] + \mathrm{h.c.}\right\} \\
&+\bar{\beta}'_{4X}\left\{\mathrm{Tr}\left[\left(\xi H^{\bar{Q}}\right)_q (1-\gamma_5)X\right] \mathrm{Tr}\left[\left(\xi^\dagger \overline{S}^Q\right)_q (1+\gamma_5)X\right] + \mathrm{h.c.}\right\} \\
&+\beta''_{4X}\mathrm{Tr}\left[\left(\xi\overline{S}^Q\right)_q (1-\gamma_5)X\right] \mathrm{Tr}\left[\left(\xi^\dagger S^{\bar{Q}}\right)_q (1+\gamma_5)X\right] \\
&+\bar{\beta}''_{4X}\mathrm{Tr}\left[\left(\xi S^{\bar{Q}}\right)_q (1-\gamma_5)X\right] \mathrm{Tr}\left[\left(\xi^\dagger \overline{S}^Q\right)_q (1+\gamma_5)X\right] . \quad (4.11)
\end{aligned}
$$

After evaluating the traces in eqs. (4.9), and keeping in mind that the external states are the pseudoscalar mesons, we have

$$
\begin{aligned}
\widetilde{O}_1 = 4\widehat{\beta}_1 &\left[\left(\xi\overline{P}^{Q*\mu}\right)_q \left(\xi P_\mu^{\bar{Q}\,*}\right)_q + \left(\xi\overline{P}^Q\right)_q \left(\xi P^{\bar{Q}}\right)_q\right] \\
&+4\widehat{\beta}'_1\left[\left(\xi\overline{P}^Q\right)_q \left(\xi P_0^{\bar{Q}}\right)_q + \left(\xi\overline{P}_0^Q\right)_q \left(\xi P^{\bar{Q}}\right)_q\right] + 4\widehat{\beta}''_1 \left(\xi\overline{P}_0^Q\right)_q \left(\xi P_0^{\bar{Q}}\right)_q , \quad (4.12)
\end{aligned}
$$

where $\widehat{\beta}_1^{(',')}$ have forms analogous to the ones written in eq. (3.4). In addition, in eqs. (4.9), (4.10) and (4.11), the fields S_q^Q and $S_q^{\bar{Q}}$ are defined in a way similar to eq. (3.2).

The corresponding tree and the 1-loop chiral diagrams are shown in figure 4. Since the couplings of the four-quark operators to the scalar meson are proportional to $\beta_i^{',''}$ and of the pseudoscalar decay constant to α^+, the cancellation between the chiral loop corrections in the operators $\widetilde{O}_i$ and in the decay constant is not automatic. For that reason, instead of writing the chiral logarithmic corrections to the bag-parameter, we will write them for
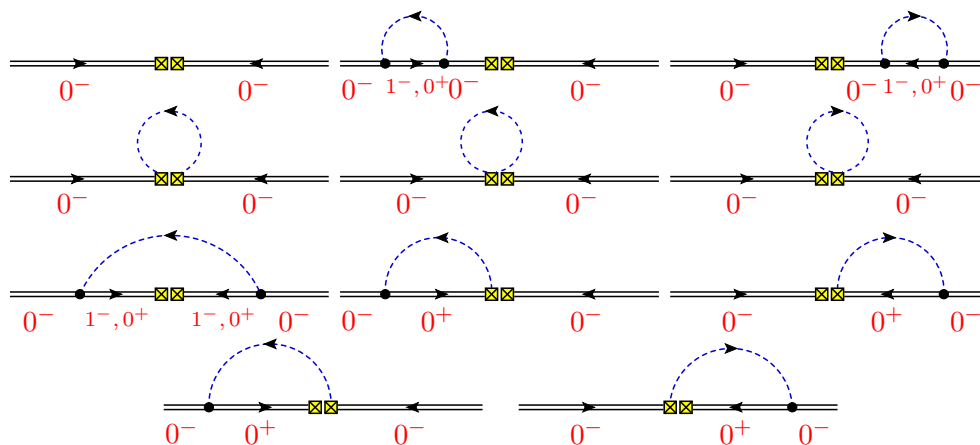
Figure 4: All diagrams which enter in the calculation of the chiral corrections to the operators $\langle \widetilde{\mathcal{O}}_{1,2,4} \rangle$. Note that we indicate when either the vector or scalar meson can propagate in the loop by $1^-, 0^+$, in contrast to the cases when only the scalar (0^+) meson can couple.

the full operator, namely

$$
\begin{aligned}
\widetilde{B}_{1q}\hat{f}_q^2 &= \frac{3}{2}\widehat{\beta}_1 \Bigg\{ 1 - \frac{t_{qa}^i t_{aq}^{i\dagger}}{2(4\pi f)^2} \Bigg[-6g^2 \lim_{x \to 0} \frac{d}{dx}[x J_1(m_i^2, x)] + 2I_1(m_i^2) \\
&\quad + 2h^2 \Big(J_1(m_i^2, \Delta_S) + J_2(m_i^2, \Delta_S) + \Delta_S \frac{d}{d\Delta_S} \big(J_1(m_i^2, \Delta_S) + J_2(m_i^2, \Delta_S) \big) \Big) \\
&\quad + 4h \frac{\widehat{\beta}_1'}{\widehat{\beta}_1} \big(I_1(m_i^2) + I_2(m_i^2, \Delta_S) \big) \Bigg] - \frac{t_{qq}^i t_{qq}^{i\dagger}}{2(4\pi f)^2} \Bigg[6g^2 \lim_{x \to 0} \frac{d}{dx}[x J_1(m_i^2, x)] + 2I_1(m_i^2) \\
&\quad + 4h \frac{\widehat{\beta}_1'}{\widehat{\beta}_1} \big(I_1(m_i^2) + I_2(m_i^2, \Delta_S) \big) + h^2 \frac{\widehat{\beta}_1''}{\widehat{\beta}_1} \sum_{k=1,2; s=\pm 1} J_k(m_i^2, s\Delta_S) \Bigg] \Bigg\}, \quad (4.13)
\end{aligned}
$$

To keep the above expression simpler we do not write the counter-terms since their structure remains the same as before, i.e., a constant times m_q and an another constant times $\mathrm{tr}\mathcal{M}$. The similar formulae for $\widetilde{B}_{2,4q}\hat{f}_q^2$ are lengthy and we will not write them explicitly. For the point that we want to make in this section it is enough to consider eq. (4.13) because in the expressions for $\widetilde{B}_{2,4q}\hat{f}_q^2$ the loop functions $I_{1,2}$ and $J_{1,2}$ occur in the same form as in eq. (4.13). In the evaluation of the sunset diagrams we used the standard simplification-trick

$$
\frac{1}{(vp - \Delta)(vp - \Delta')} = \frac{1}{\Delta - \Delta'} \left(\frac{1}{vp - \Delta} - \frac{1}{vp - \Delta'} \right). \quad (4.14)
$$

We now turn to the case $m_\pi \ll \Delta_S$ and study the behavior of eq. (4.13) around $m_\pi^2 \to 0$. In addition to the limits discussed in eq. (4.6), when dealing with the integrals in the last line of eq. (4.13) we shall proceed similarly to what has been done in ref. [31],

namely we expand the integrand in E_π/Δ_S and write

$$\sum_{k=1,2;s=\pm1} J_k(m_\pi^2, s\Delta_S) = -2(4\pi)^2 v_\mu v_\nu \times i\mu^\epsilon \int \frac{d^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu}{(p^2 - m_\pi^2)[\Delta_S^2 - (vp)^2]}$$

$$= -\frac{2(4\pi^2)}{\Delta_S^2} v_\mu v_\nu \left[i\mu^\epsilon \int \frac{d^{4-\epsilon}p}{(2\pi)^{4-\epsilon}} \frac{p^\mu p^\nu}{p^2 - m_\pi^2} + \mathcal{O}(1/\Delta_S^2) \right]$$

$$= -\frac{m_\pi^2}{2\Delta_S^2}(4\pi^2) I_1(m_\pi^2) + \cdots \rightarrow -\frac{m_\pi^4}{2\Delta_S^2} \log \frac{m_\pi^2}{\mu^2} + \ldots, \quad (4.15)$$

where the ellipses stand for the terms of higher order in m_π^2/Δ_S^2. Note, however, that even the leading term is already of higher order in the chiral expansion and thus the terms proportional to h in eq. (4.13) do not affect the leading chiral logarithmic corrections.

On the basis of the above discussion and eqs. (4.5), (4.6) we see that after expanding eq. (4.13) around $m_\pi^2 = 0$, the leading chiral logarithms arising from the pion loops remain unchanged even when the coupling to the scalar meson is included in the loops. On the other hand, as discussed in the previous subsection, the logarithms arising from the kaon and the η-meson are competitive in size with those arising from the coupling to the heavy-light scalar meson, which is the consequence of the smallness of Δ_S. Therefore, like for the decay constants, the relevant chiral expansion is the one derived in the $SU(2)_L \otimes SU(2)_R \rightarrow SU(2)_V$ theory, i.e.,

$$\widetilde{B}_{1q} \hat{f}_q^2 = \widetilde{B}_1^{\text{Tree}} \alpha^2 \left[1 - \frac{3g^2 + 2}{(4\pi f)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_{\mathcal{O}_1}(\mu) m_\pi^2 \right],$$

$$\widetilde{B}_{2,4q} \hat{f}_q^2 = \widetilde{B}_{2,4}^{\text{Tree}} \alpha^2 \left[1 - \frac{3g^2(3 - Y) + 3 \pm 1}{2(4\pi f)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_{\mathcal{O}_{2,4}}(\mu) m_\pi^2 \right], \quad (4.16)$$

or by using eq. (4.7), for the bag parameters we obtain

$$\widetilde{B}_{1q} = \widetilde{B}_1^{\text{Tree}} \left[1 - \frac{1 - 3g^2}{2(4\pi f)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_{B_1}(\mu) m_\pi^2 \right], \quad (4.17)$$

$$\widetilde{B}_{2,4q} = \widetilde{B}_{2,4}^{\text{Tree}} \left[1 + \frac{3g^2 Y \mp 1}{2(4\pi f)^2} m_\pi^2 \log \frac{m_\pi^2}{\mu^2} + c_{B_{2,4}}(\mu) m_\pi^2 \right], \quad (4.18)$$

which coincide with the pion loop contributions shown in eqs. (3.11) and (3.12), as they should.

5. Relevance to the analyses of the lattice QCD data

It should be stressed that the consequence of the discussion in the previous section is mainly important to the phenomenological approaches in which the sizable kaon and η-meson logarithmic corrections are taken as predictions, whereas the counter-term coefficients are fixed by matching to large N_c expansion or some other model. We showed that the contributions of the nearby heavy-light scalar states are competitive in size and thus they cannot be ignored nor separated from the discussion of the kaon and/or η-meson loops.

In the extrapolation of the lattice data, instead, this is not a problem because the kaon and the η-meson loops essentially do not alter the quark mass dependence, whereas the
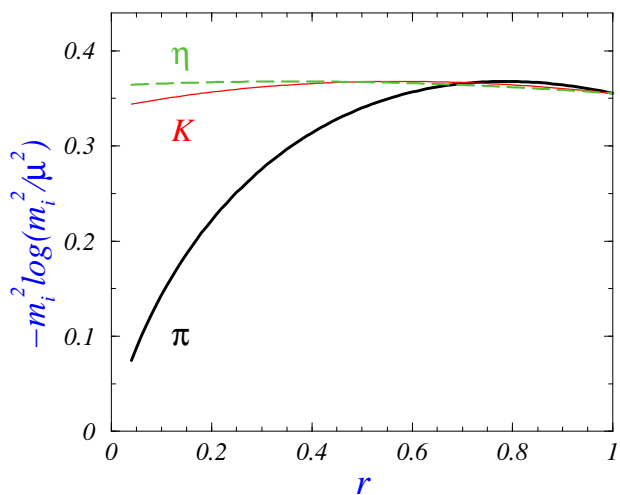
Figure 5: Typical chiral logarithmic contributions $-m_i^2 \log(m_i^2/\mu^2)$ are shown for pion, kaon and η as a function of $r = m_d/m_s$, with m_s fixed to its physical value, and $\mu = 1\,\mathrm{GeV}$.

important nonlinearity comes from the pion chiral loops. As an illustration, in figure 5 we plot the typical chiral logarithm, $-m_i^2 \log(m_i^2/\mu^2)$, as a function of $r = m_d/m_s$ which appear in the Gell-Mann-Oakes-Renner formulae,

$$m_\pi^2 = 2B_0 m_s r \,, \quad m_K^2 = 2B_0 m_s \frac{r+1}{2} \,, \quad m_\eta^2 = 2B_0 m_s \frac{r+2}{3} \,, \tag{5.1}$$

with $2B_0 m_s = 2m_K^2 - m_\pi^2 = 0.468\ \mathrm{GeV}^2$. Thus the fact that the nearby scalar heavy-light mesons do not spoil the pion logarithmic correction to the decay constant and the bag-parameters is most welcome from the lattice practitioners' point of view, because the formulae derived in HMChPT can still (and should) be used to guide the chiral extrapolations of the lattice results, albeit for the pion masses lighter than Δ_S.

6. Conclusion

In this paper we revisited the computation of the $B_q^0 - \overline{B}_q^0$ mixing amplitudes in the framework of HMChPT. Besides the SM bag parameter, we also provided the expressions for the chiral logarithmic correction to the SUSY bag parameters. More importantly, we study the impact of the near scalar mesons to the predictions derived in HMChPT in which these contributions were previously ignored. We showed that while the corrections due to the nearness of the scalar mesons are competitive in size with the kaon and η meson loop corrections, they do not alter the pion chiral logarithms. In other words the valid (pertinent) ChPT expressions for the quantities discussed in this paper are those involving pions only. This is of major importance for the chiral extrapolations of the results obtained from the QCD simulations on the lattice, because precisely the pion chiral logarithms provide the important guidance in those extrapolations as long as $m_\pi \ll \Delta_S$. The corresponding useful formulas are given in eqs. (4.7), (4.17), (4.18). As a side-result

we verified that the chiral logarithmic corrections to the scalar meson decay constant are the same as to the pseudoscalar one, modulo replacement $g \to \widetilde{g}$ (c.f. eq. (4.8)).

Acknowledgments

References

[1] UA1 collaboration, C. Albajar et al., *Search for $B^0 - \bar{B}^0$ oscillations at the CERN proton-anti-proton collider. 2*, *Phys. Lett.* **B 186** (1987) 247 [*Erratum ibid.* **197** (1987) 565].

[2] CDF AND D0 collaboration, D. Whiteson, *Precision measurements of the top quark mass at the Tevatron*, hep-ex/0605106.

[3] HEAVY FLAVOR AVERAGING GROUP (HFAG) collaboration, E. Barberio et al., *Averages of b-hadron properties at the end of* 2005, hep-ex/0603003.

[4] CDF - RUN II collaboration, A. Abulencia et al., *Measurement of the B_s^0 - $\bar{B}_s^0$ oscillation frequency*, *Phys. Rev. Lett.* **97** (2006) 062003 [hep-ex/0606027].

[5] UTFIT collaboration, M. Bona et al., *The unitarity triangle fit in the standard model and hadronic parameters from lattice QCD: a reappraisal after the measurements of ΔM_s and $BR(B \to \tau \nu_\tau)$*, *JHEP* **10** (2006) 081 [hep-ph/0606167];
CKMFITTER GROUP collaboration, J. Charles et al., *CP-violation and the CKM matrix: assessing the impact of the asymmetric B factories*, *Eur. Phys. J.* **C 41** (2005) 1 [hep-ph/0406184].

[6] T. Onogi, *Heavy flavor physics from lattice QCD*, PoS(LAT2006)017 [hep-lat/0610115];
M. Wingate, *B physics on the lattice: present and future*, *Mod. Phys. Lett.* **A 21** (2006) 1167 [hep-ph/0604254];
S. Hashimoto, *Recent results from lattice calculations*, *Int. J. Mod. Phys.* **A 20** (2005) 5133 [hep-ph/0411126];
D. Becirevic, *Status of the computation of $f_{B_{s,d}}$, ξ and $\widehat{g}$*, hep-ph/0310072.

[7] JLQCD collaboration, N. Yamada et al., *B meson B-parameters and the decay constant in two-flavor dynamical QCD*, *Nucl. Phys.* **106** *(Proc. Suppl.)* (2002) 397 [hep-lat/0110087];
A.S. Kronfeld and S.M. Ryan, *Remark on the theoretical uncertainty in $B^0 - \bar{B}^0$ mixing*, *Phys. Lett.* **B 543** (2002) 59 [hep-ph/0206058];
D. Becirevic, S. Fajfer, S. Prelovsek and J. Zupan, *Chiral corrections and lattice QCD results for f_{B_s}/f_{B_d} and $\Delta m_{B_s}/\Delta m_{B_d}$*, *Phys. Lett.* **B 563** (2003) 150 [hep-ph/0211271].

[8] R. Casalbuoni et al., *Phenomenology of heavy meson chiral lagrangians*, *Phys. Rept.* **281** (1997) 145 [hep-ph/9605342];
A. V. Manohar and M.B. Wise, *Heavy quark physics*, *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.* **10**, 1 (2000);
J.F. Donoghue, E. Golowich and B.R. Holstein, *Dynamics of the standard model*, *Camb. Mon. Part. Phys. Nucl. Phys. Cosmol.* **2** (1992) 1.

[9] S. Descotes-Genon, N.H. Fuchs, L. Girlanda and J. Stern, *Resumming QCD vacuum fluctuations in three-flavour chiral perturbation theory*, *Eur. Phys. J.* **C 34** (2004) 201 [`hep-ph/0311120`]; *Chiral order and fluctuations in multi-flavour QCD*, *Eur. Phys. J.* **C 27** (2003) 115 [`hep-ph/0207337`].

[10] A. Manohar and H. Georgi, *Chiral quarks and the nonrelativistic quark model*, *Nucl. Phys.* **B 234** (1984) 189.

[11] A. Pich, *Chiral perturbation theory*, *Rept. Prog. Phys.* **58** (1995) 563 [`hep-ph/9502366`].

[12] J. Gasser and H. Leutwyler, *Chiral perturbation theory: expansions in the mass of the strange quark*, *Nucl. Phys.* **B 250** (1985) 465.

[13] BABAR collaboration, B. Aubert et al., *Observation of a narrow meson decaying to $D_s^+ \pi^0$ at a mass of $2.32\,GeV/c^2$*, *Phys. Rev. Lett.* **90** (2003) 242001 [`hep-ex/0304021`];
CLEO collaboration, D. Besson et al., *Observation of a narrow resonance of mass $2.46\,GeV/c^2$ decaying to $D_s^{*+}\pi_0$ and confirmation of the $D_{sJ}^*(2317)$ state*, *Phys. Rev.* **D 68** (2003) 032002 [`hep-ex/0305100`];
K. Abe et al., *Measurements of the $D_s J$ resonance properties*, *Phys. Rev. Lett.* **92** (2004) 012002 [`hep-ex/0307052`];
FOCUS collaboration, E.W. Vaandering, *Charmed hadron spectroscopy from focus*, `hep-ex/0406044`.

[14] BELLE collaboration, K. Abe et al., *Study of $B^- \to D^{**0}\pi^-$ $(D^{**0} \to D^{(*)+}\pi^-)$ decays*, *Phys. Rev.* **D 69** (2004) 112002 [`hep-ex/0307021`];
FOCUS collaboration, J.M. Link et al., *Measurement of masses and widths of excited charm mesons D_2^* and evidence for broad states*, *Phys. Lett.* **B 586** (2004) 11 [`hep-ex/0312060`].

[15] P. Colangelo, F. De Fazio and R. Ferrandes, *Excited charmed mesons: observations, analyses and puzzles*, *Mod. Phys. Lett.* **A 19** (2004) 2083 [`hep-ph/0407137`];
E.S. Swanson, *The new heavy mesons: a status report*, *Phys. Rept.* **429** (2006) 243 [`hep-ph/0601110`];
S. Godfrey, *Properties of the charmed P-wave mesons*, *Phys. Rev.* **D 72** (2005) 054029 [`hep-ph/0508078`];
D. Becirevic, S. Fajfer and S. Prelovsek, *On the mass differences between the scalar and pseudoscalar heavy-light mesons*, *Phys. Lett.* **B 599** (2004) 55 [`hep-ph/0406296`].

[16] H.-W. Lin, S. Ohta, A. Soni and N. Yamada, *Charm as a domain wall fermion in quenched lattice QCD*, *Phys. Rev.* **D 74** (2006) 114506 [`hep-lat/0607035`].

[17] UKQCD collaboration, A.M. Green, J. Koponen, C. McNeile, C. Michael and G. Thompson, *Excited B mesons from the lattice*, *Phys. Rev.* **D 69** (2004) 094505 [`hep-lat/0312007`].

[18] D. Becirevic et al., *$B_d - \bar{B}_d$ mixing and the $B_d \to J_\psi K_S$ asymmetry in general SUSY models*, *Nucl. Phys.* **B 634** (2002) 105 [`hep-ph/0112303`];
P. Ball, S. Khalil and E. Kou, *$B_s^0 - \bar{B}_s^0$ mixing and the $B_s \to J_\psi \Phi$ asymmetry in supersymmetric models*, *Phys. Rev.* **D 69** (2004) 115011 [`hep-ph/0311361`];
A.J. Buras, P.H. Chankowski, J. Rosiek and L. Slawianowska, *$\Delta M_{B_{d,s}}$, $B_{d,s}^0 \to \mu^+\mu^-$ and $B \to X_s\gamma$ in supersymmetry at large $\tan\beta$*, *Nucl. Phys.* **B 659** (2003) 3 [`hep-ph/0210145`];
V. Barger, C.-W. Chiang, J. Jiang and P. Langacker, *$B_s - \bar{B}_s$ mixing in Z' models with flavor-changing neutral currents*, *Phys. Lett.* **B 596** (2004) 229 [`hep-ph/0405108`];
P. Ball and R. Fleischer, *Probing new physics through B mixing: status, benchmarks and prospects*, *Eur. Phys. J.* **C 48** (2006) 413 [`hep-ph/0604249`].

[19] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A complete analysis of fcnc and CP constraints in general SUSY extensions of the standard model*, *Nucl. Phys.* **B 477** (1996) 321 [`hep-ph/9604387`].

[20] R. Sommer, *Non-perturbative QCD: renormalization, O(a)-improvement and matching to heavy quark effective theory*, `hep-lat/0611020`;
ALPHA collaboration, P. Dimopoulos et al., *Non-perturbative scale evolution of four-fermion operators in two-flavour QCD*, `hep-lat/0610077`;
D. Becirevic et al., *Renormalization constants of quark operators for the non- perturbatively improved Wilson action*, *JHEP* **08** (2004) 022 [`hep-lat/0401033`].

[21] D. Becirevic and G. Villadoro, *Remarks on the hadronic matrix elements relevant to the SUSY $K_0 - \bar{K}_0$ mixing amplitude*, *Phys. Rev.* **D 70** (2004) 094036 [`hep-lat/0408029`].

[22] W. Detmold and C.J.D. Lin, *Matrix elements of the complete set of $\Delta B = 2$ and $\Delta C = 2$ operators in heavy meson chiral perturbation theory*, `hep-lat/0612028`.

[23] S.R. Sharpe, *Applications of chiral perturbation theory to lattice QCD*, `hep-lat/0607016`;
C. Aubin, *Current physics results from staggered chiral perturbation theory*, *Mod. Phys. Lett.* **A 21** (2006) 2913 [`hep-lat/0612013`];
C. Bernard et al., *Low energy constants from the MILC collaboration*, `hep-lat/0611024`.

[24] D. Becirevic, S. Prelovsek and J. Zupan, *$B \to \pi$ and $B \to K$ transitions in standard and quenched chiral perturbation theory*, *Phys. Rev.* **D 67** (2003) 054010 [`hep-lat/0210048`].

[25] B. Grinstein, E. Jenkins, A.V. Manohar, M.J. Savage and M.B. Wise, *Chiral perturbation theory for f_{D_s}/f_D and B_{B_s}/B_{B_d}*, *Nucl. Phys.* **B 380** (1992) 369 [`hep-ph/9204207`].

[26] A. Hiorth and J.O. Eeg, *Non-factorizable effects in $B^0 - \bar{B}^0$ mixing*, *Eur. Phys. J. direct* **C30** (2003) 006 [*Eur. Phys. J.* **C 32** (2004) 69] [`hep-ph/0301118`].

[27] D. Arndt and C.J.D. Lin, *Heavy meson chiral perturbation theory in finite volume*, *Phys. Rev.* **D 70** (2004) 014503 [`hep-lat/0403012`].

[28] T. Mehen and R.P. Springer, *Even- and odd-parity charmed meson masses in heavy hadron chiral perturbation theory*, *Phys. Rev.* **D 72** (2004) 034006 [`hep-ph/0503134`].

[29] UKQCD collaboration, G. Herdoiza, C. McNeile and C. Michael, *Decay constants of P-wave heavy-light mesons from unquenched lattice QCD*, *Phys. Rev.* **D 74** (2006) 014510 [`hep-lat/0604001`];
UKQCD collaboration, C. McNeile and C. Michael, *Searching for chiral logs in the static-light decay constant*, *JHEP* **01** (2005) 011 [`hep-lat/0411014`].

[30] P. Colangelo and F. De Fazio, *QCD interactions of heavy mesons with pions by light-cone sum rules*, *Eur. Phys. J.* **C 4** (1998) 503 [`hep-ph/9706271`];
Y.-B. Dai, C.-S. Huang, M.-Q. Huang, H.-Y. Jin and C. Liu, *Decay widths of excited heavy mesons from QCD sum rules in the leading order of HQET*, *Phys. Rev.* **D 58** (1998) 094032 [`hep-ph/9705223`];
J. Lu, X.-L. Chen, W.-Z. Deng and S.-L. Zhu, *Pionic decays of $D_{sJ}(2317)$, $D_{sJ}(2460)$ and $B_{sJ}(5718)$, $B_{sJ}(5765)$*, *Phys. Rev.* **D 73** (2006) 054012 [`hep-ph/0602167`];
D. Becirevic et al., *Pionic couplings $\widehat{g}$ and $\widetilde{g}$ in the static heavy quark limit*, `PoS(LAT2005)212` [`hep-lat/0510017`].

[31] S. Fajfer and J. Kamenik, *Chiral loop corrections to strong decays of positive and negative parity charmed mesons*, *Phys. Rev.* **D 74** (2006) 074023 [`hep-ph/0606278`].